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THE PROBABILISTIC ORIGIN OF BELL'S INEQUALITY

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Abstract

The concept of local realism entails certain restrictions concerning the possible occurrence of correlated events. Although these restrictions are inherent in classical physics they have never been noticed until Bell has shown in 1964 that in general correlations in quantum mechanics can not be interpreted in a classical way. We demonstrate how a local realistic way of thinking about measurement results necessarily leads to limitations with regard to the possible appearance of correlated events. These limitations, which are equivalent to Bell's inequality can be easily formulated as an immediate consequence of our discussion.

1 Introduction

Local realism denotes a certain way of thinking about the origin of experimental results which can be specified by the concepts of *locality* and *reality* as defined in the EPR paper [1]. For a system consisting of two spatially separated parts (e.g. in a singlet state) *locality* means that, "since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system." As a criterion for *reality* EPR give a reasonable proposition which reads as follows: "If,without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

Generalizing the EPR reality criterion in such a way that with regard to the singlet state (Bohm's version) the result of any spin-measurement has to be considered as predetermined, Bell has shown in 1964 [2] that local realism as defined above is "incompatible with the statistical predictions of quantum mechanics."

By applying the concepts of local realism to a three particle system, D.Greenberger,M.Horne and A.Zeilinger [3] (see also [4]) have shown that in this case a clear cut contradiction can be deduced on the level of perfect correlations. Although this approach provides the most expressive demonstration of the incompatibility of local realism with quantum mechanics it does not work for two-particle systems. There the incompatibility arises just on the statistical level. Hence an intuitive understanding of the contradiction between the idea of local realism and quantum mechanics is difficult, if one is not aware of the origin of this contradiction.

One attempt in order to demonstrate the basic idea of Bell's proof in a more expressive way has been made by E.P.Wigner [5], who derived a specific form of Bell's inequality by using only simple settheoretical arguments. Recently another attempt has been made by Lucien Hardy [6] who showed that the probability for a contradiction of the GHZ-kind can be greater than zero for a two-particle system.

Nevertheless none of these approaches has provided a general argument based on the concepts of locality and reality which explicitly demonstrates the origin of the discrepancy between local realism and quantum mechanics. Thus our aim is to show the essential restrictions of local realism by discussing the results of a general two-particle experiment using the assumptions of *locality* and *reality*. As evident consequences the conditions for the fulfilment of these restrictions are equivalent to Bell's inequality.

2 Predictions based on the knowledge of correlations

We consider the following experimental setup (cf. Fig. 1): A source emits the two parts of a system in opposite directions. Measurements with the possible results +1 and −1 are performed on each part by two observers A and B. Each of them may select one of two possible values of a measurement parameter α and β, respectively. As a consequence four different experiments can be made, corresponding to the four different combinations of the measurement parameters α_1, α_2 and β_1, β_2.

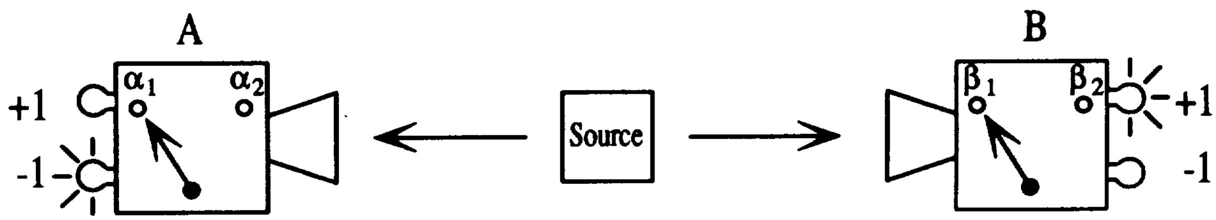

FIG. 1. The experimental setup consists of a source which emits the two parts of a system in opposite directions. Measurements with the possible results +1 and −1 are performed on each part by two observers A and B. Both A and B have a knob which selects one of two possible values of a measurement parameter. In such a way 4 different experiments can be made (cf. table I).

We assume that all four experiments have been made. The results are listed in table I.

TABLE I. The correlations of the results of the 4 experiments are listed. They might have been observed in actual experiments or calculated by quantum mechanics. $P_i^{\neq}$ is the probability for different results in experiment i. In consequence the probability for equal results $P_i^{=}$ is $1 - P_i^{\neq}$.

Experiment	A	B	Correlation
1	α_1	β_1	$P_1^{\neq}$
2	α_1	β_2	$P_2^{\neq}$
3	α_2	β_1	$P_3^{\neq}$
4	α_2	β_2	$P_4^{\neq}$

In experiment 1 with the parameters adjusted to α_1 and β_1 the observers A and B got different results with probability $P_1^{\neq}$. In experiments 2, 3 and 4 the probabilities for different results are $P_2^{\neq}$, $P_3^{\neq}$ and $P_4^{\neq}$, respectively.

Knowing the correlations which have to be expected either from previous experiments or quantum mechanical calculations, A is able to predict the possible results of B and vice versa. This means that after A has for example performed a series of n measurements with the setting α_1 he can infer all possible results of B on the basis of his experimental data and the knowledge of the correlations in experiments 1 and 2 by the following reasoning.

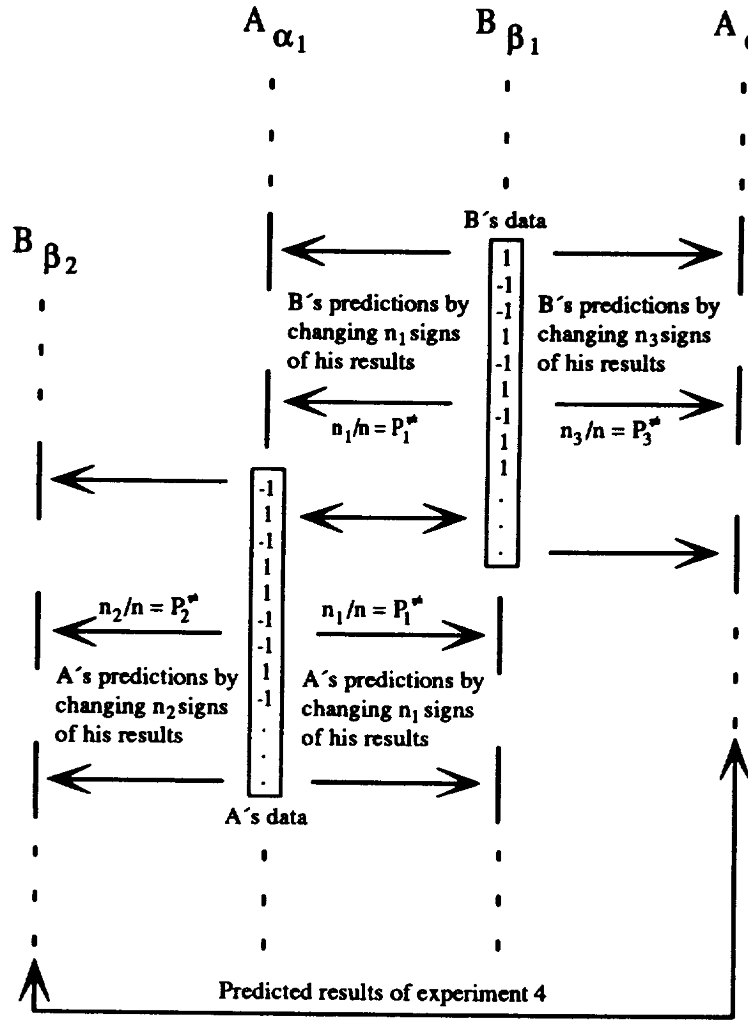

FIG. 2. A procedure is shown by which observer A (B) after having performed a series of n measurements is able to predict the results of observer B (A) for the two alternative settings of the measurement parameter β (α). The actually measured results of observers A and B are listed in the two boxes. By changing a corresponding number of signs ($n_i = P_i^{\neq} \cdot n$ for experiment i) of the measured results the predictions are in agreement with the calculated or previously observed correlations listed in table I. Nevertheless it turns out immediately that the predicted results of experiment 4 are consistent with the actual correlations of experiment 4 (cf. table I) **only if** the inequality $\boldsymbol{n_1 + n_2 + n_3 \geq n_4}$ (equivalent to Bell's inequality) is fulfilled.

If B selects the parameter β_1 (experiment 1) the probability for different results has to be $P_1^{\neq}$ (cf. table I). For $n \to \infty$ this means that $n_1 = P_1^{\neq} \cdot n$ results have to be different. By reversing n_1 signs of his results A arrives at a series of possible results of B which correspond to the known correlations. Because there are $\frac{n!}{n_1!(n-n_1)!}$ different ways of changing n_1 signs of n numbers A ends up with a list of $\frac{n!}{n_1!(n-n_1)!}$ different predictions for the possible outcomes of B's measurements. In the same way A is able to infer the possible results B could get, if B selects β_2 (experiment 2) by changing $n_2 = P_2^{\neq} \cdot n$ signs of his results. In Figure 2 the results of A are listed in the box in row A_{α_1}. The predictions he derives from these data are symbolized by vertical lines in rows B_{β_1} and B_{β_2}. Each line corresponds to one way of changing n_1 and n_2 signs, respectively. By this means the predicted results correspond to the known correlations.

Now let's assume that observer B actually selects the parameter β_1 and performs a series of n measurements. In figure 2 his results are listed in the box in row B_{β_1}. Of course they correspond to one of the predictions by A.

Not knowing what A has done observer B himself predicts all possible results A could get if he selects the parameter α_1 (experiment 1) or α_2 (experiment 3) (cf. table I) by considering all possible ways of changing n_1 or n_3 signs of his results. Again the actual results of A correspond to one of the predictions by B as it is shown in figure 2.

3 Bell's inequality

In the previous section we have shown how it is possible for A to predict all results B could obtain and vice versa. In the following we are going to apply the *locality* assumption that "no real change can take place in the second system in consequence of anything that may be done to the first system" [1]. Moreover we assume in agreement with *realistic* approaches that "unperformed experiments **have** results" [7] or in other words that predicted results have the status of potential reality.

If we now ask what A could have measured if he had selected the parameter α_2 (experiment 3) instead of α_1 (experiment 1), we just have to take into consideration the predictions by observer B to find the answer. Based on his actual results and the known correlation in experiment 3 (cf. table I) observer B has predicted all results A could have got if he had chosen α_2 (cf. figure 2). As a consequence of the *locality* assumption the results of B, which are the basis of his predictions, are *independent* of anything that may be done by A. Because of this independence *all* of B's predictions have the status of potential reality, which means that if A had selected α_2 he actually would have got one of the results predicted by B.

In the same way we find the answer to the question what B could have measured if he had selected the parameter β_2 (experiment 2) instead of β_1 (experiment 1) by considering the predictions by observer A (cf. figure 2). It is important to notice that because of the *locality* assumption we can make independent use of the predictions by B and A to answer the question what A and B could have measured if they had selected α_2 and β_2, respectively.

Since we know all possible results A and B could have got if they had chosen α_2 and β_2, respectively (experiment 4), we may now try to find out if these results are consistent with the known correlation of experiment 4 $P_4^{\neq}$ (cf. table I). For this purpose we take one of the results B could have got if he had selected β_2 (row B_{β_2} in figure 2), change n_2 signs to get the actual results of A (box in row A_{α_1}), change n_1 signs to get the actual results of B (box in B_{β_1}) and change n_3 signs to end up with one of the results A could have got if he had selected α_2 (row A_{α_2} in figure 2). Of course we could also do the same thing the other way round but anyway the results A could have got are connected to the results B could have got by the following transformation rule (cf. figure 2): Reverse n_2 signs in the first, n_1 signs in the second and n_3 signs in the last step or the other way round. Doing this the **maximum number** of signs one can change is simply $\boldsymbol{n_1 + n_2 + n_3}$. This result of *local realistic* reasoning is consistent with the observed correlation in experiment 4 if and only if $\boldsymbol{n_1 + n_2 + n_3 \geq n_4} = \mathrm{P}_4^{\neq} \cdot \boldsymbol{n}$. If this condition is violated, then not a single pair of the predicted results of experiment 4 (cf. figure 2) is correlated in agreement with experience because there is no pair with more than $n_1 + n_2 + n_3$ different signs.

It follows immediately that this condition is equivalent to Bell's inequality:

$$\mathrm{n}_1 + \mathrm{n}_2 + \mathrm{n}_3 \geq \mathrm{n}_4$$

$$\Big\Downarrow \frac{n_i}{n} = \mathrm{P}_i^{\neq} \qquad i = 1,2,3,4$$

$$\mathrm{P}_1^{\neq} + \mathrm{P}_2^{\neq} + \mathrm{P}_3^{\neq} \geq \mathrm{P}_4^{\neq} \tag{1}$$

$$\Big\Downarrow \mathrm{P}_i^{\neq} + \mathrm{P}_i^{=} = 1 \qquad i = 1,2,3,4$$

$$1 - \mathrm{P}_1^{=} + 1 - \mathrm{P}_2^{=} + 1 - \mathrm{P}_3^{=} \geq 1 - \mathrm{P}_4^{=} \tag{2}$$

$$\Big\Downarrow \mathrm{E}_i = \mathrm{P}_i^{=} - \mathrm{P}_i^{\neq} \qquad i = 1,2,3,4$$

$$\boxed{\mathrm{E}_1 + \mathrm{E}_2 + \mathrm{E}_3 - \mathrm{E}_4 \leq 2} \tag{3}$$

We get (3) by adding inequalities (1) and (2) and using the definition of the expectation value of the product of the results E_i in experiment i ($i = 1,2,3,4$).

4 Discussion

We have shown that just by discussing the possible results of a general two-particle experiment in a local realistic way one is directly led to a condition for the consistency between quantum mechanics and the concept of local realism.

The crucial point in the argumentation is on the one hand the assumption that A's and B's data are determined locally, which means that A's (B's) results are completely independent of the measurement parameter selected by B (A). On the other hand by assuming that unperformed experiments have results A's and B's predictions can be combined in order to get a prediction of experiment 4 (unperformed). It turns out that this kind of counterfactual reasoning is inconsistent with the results one obtains by actually making experiment 4.

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References

[1] A.Einstein,B.Podolsky, and N.Rosen,"Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" Phys.Rev.**47**,777-780(1935).

[2] John S.Bell,"On the Einstein Podolsky Rosen Paradox" *Physics* 1,195-200(1964)

[3] D.M.Greenberger,M.Horne,and A.Zeilinger,"Going beyond Bell's theorem",in *Bell's Theorem,Quantum Theory,and Concepts of the Universe*,edited by M.Kafatos (Kluwer Academic,Dordrecht,The Netherlands,1989) ,pp.73-76

[4] D.M.Greenberger,M.Horne,A.Shimony,and A.Zeilinger, "Bell's theorem without inequalities",-Am.J.Phys.**58**,1131-1143(1990)

[5] Eugen P.Wigner,"On Hidden Variables and Quantum Mechanical Probabilities" Am.J.Phys. **38**,1005-1009(1970)

[6] Lucien Hardy,"A New Way to Obtain Bell Inequalities" Phys.Lett. A **161**, 21-25 (1991)

[7] Asher Peres, "Unperformed experiments have no results" Am.J.Phys. **46**, 745-747 (1978)