GLOW FROM THE LIMB SIDE OF A SPHERICAL PLANETARY ATMOSPHERE

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ABSTRACT: An approximate formula is derived for calculating the diffuse radiation from the limb side of a spherical planetary atmosphere. The case of an atmosphere adjoining a surface that reflects light istropically is considered. For the case of very small optical thickness of the atmosphere a simple analytical expression is found for computation of diffuse radiation intensity in the assumption that scattering coefficient of the aerosol and gas atmospheric components changes exponentially with altitude. This formula is used for calculation of the limbside radiation intensity of the Martian atmosphere (Figures 2-4).

The glow of the layer of atmosphere directly beyond the limb of a planet's natural surface can be observed when investigating planets from deep space at sufficient close range on the limb side. We understand the planet's natural surface to mean its hard surface (Mars, Earth when there are no clouds) or its cloud cover (Venus, Earth, Jupiter). In many cases it is necessary to take into consideration the distribution of brightness along the above-indicated layer, assigning optical parameters to the atmosphere and to the planetary surface. Then, by comparing the observed distribution of the brightness with the calculated we can determine, for example, the significance of a characteristic such as the change in the concentration of aerosol with altitude for atmospheric optics, for this, in the final analysis, will enable us to build an optical model of the atmosphere. /55

We cannot assume the atmosphere to be plane-parallel when estimating the distribution of brightness along a planet's atmospheric layers, but should take its sphericity into consideration. This is connected with the fact that in the case cited the radiation comes to the observer from a layer of atmosphere, the extent of which is comparable to the radius of the planet. The equation for the transfer of radiation in a spherical atmosphere is described in reference [1]. I. N. Minin and V. V. Sobolev [2] derived the equation for the source function, $B(r, \psi)$ and (r and ψ are the spherical coordinates of the point), for a symmetrical spherical planetary atmosphere illuminated by parallel rays for the case of isotropic scattering[4]. Considered in this same source is the case

4. The problem reviewed is axisymmetrical because the axis is a straight line joining the centers of the planet and the sun.

when the atmosphere is contiguous to an isotropically reflecting surface with specified albedo, A. These same authors, in their articles [3-5], used the approximation method to reduce the problem of diffuse reflection and light transmission by a symmetrical spherical planetary surface to a second order partial differential equation for the corresponding boundary conditions. Numerical calculations for selected special cases were made by way of examples.

Following V. V. Sobolev and I. N. Minin was O. I. Smoktiy [6, 7], who derived an approximate analytical solution of the problem of multiple scattering of light in a spherical atmosphere thus making possible effective numerical calculations, and calculated the intensity of the outgoing radiation for different models of the vertical structure of a symmetrical spherical atmosphere.

The calculation for brightness distribution along the planet's limb side atmospheric layers can be made using the methods in references [3-7]. They are, however, rather complicated when repeated light scattering must be taken into consideration. This paper presents a very simple approximation method for calculating this distribution when the line of sight does not intersect the plane of the terminator. And it is assumed that the atmosphere is contiguous to a surface that is reflecting isotropically. A calculation is made of the brightness distribution along the Martian atmosphere layers as observed directly beyond the limb of its natural surface, for illustrative purposes.

Accordingly, let a spherical, symmetrical atmosphere, the radius of the upper boundary of which is equal to R_A, be illuminated by parallel rays from the Sun. Let us designate the illumination of a unit area at the upper boundary of the atmosphere perpendicular to the incident beams by πS. Let the observer be /56 found at point K (Figure 1). Let us take the current point, T, to be on the line of sight, KB, the radius vector of which is equal to r. The position of the point T on the line of sight can be determined by the segment BT = s, read from point B. Then the intensity of the diffuse radiation, I, reaching the observer from the column of atmosphere located along the line of sight, AB, can be determined from

$$I(s_0) = \int_0^{s_0} B(s) \exp\left[\int_{s_0}^{s} \alpha(z)\, ds \right] \alpha(z)\, ds, \qquad (1)$$

where

B(s) is the source function at point T;

s_0 = AB,

$\alpha(z)$ is the absorption factor;

z is the altitude of point T above the planetary surface (r = R + z).

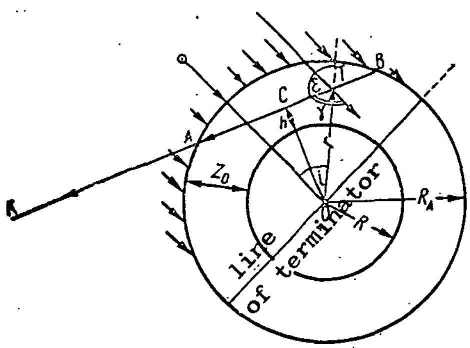

Figure 1.

The thickness of the planetary atmosphere, z_0, is very much less than the planet's radius, R, so we can say that the atmospheric layers are plane-parallel. But we can take it that the quantity of radiant energy from the Sun, and absorbed by an elementary volume of atmosphere, remains as for a spherical atmosphere. In other words, we are considering the change in illumination of the atmosphere along the line of sight, AB. Reference [2] already has used the approximation method described. Now we shall use it to write, in explicit form, an expression for the source function B(s). It is obvious that the above approximation is better justified the smaller z_0/R, and that significant disagreement is to be expected only near the terminator. We shall, therefore, limit ourselves in what follows to the case when the line of sight does not intersect the plane of the terminator within the limits of the atmosphere.

Let us designate the angle between the radius vector r for point T and the direction to the Sun by i = arc cos ζ, and the angle between r and the direction to the observer at point T by ϵ = arc cos η (Figure 1). Then let us assume that the atmosphere of the planet is purely scattering (that is, it has a single scattering albedo, $\lambda = 1$), and is contiguous to a surface with given albedo A that reflects isotropically. Then, if the atmosphere is homogeneous, the approximate expression for source function B in the plane of the atmosphere, given the arbitrary (but not too elongated) scattering indicatrix $X(\gamma)$ obtained by V. V. Sobolev (see reference [8], Chapter 10), can be used.

But atmospheric heterogeneity has a significant effect on brightness distribution along atmospheric layers. In this case, the heterogeneity of the atmosphere for first order scattering can be determined by the scattering indicatrix, $\chi(\gamma)$, in terms of altitude, z (or of optical depth, τ). So far as the component ΔB (τ, η, ζ) of the source function, determined by higher order scatterings, is concerned, in this case, as follows from the results of the generalization of the approximate formula from [8] by S. D. Gutshabash [9] we have

$$\Delta B(\tau, \eta, \zeta) = \bar{I}(\tau) + x_1(\tau)\bar{H}(\tau)\eta, \tag{2}$$

where /57

$$\bar{I}(\tau) = \frac{S\zeta}{2}\left(1+\frac{3}{2}\zeta\right) - \frac{3}{4}S\zeta^2 e^{-\tau/\zeta} - F\left[2+\left(3-\frac{1}{\tau}\int_0^\tau x_1(t)\,dt\right)\tau\right], \tag{3}$$

$$F = \frac{S}{2}\,\frac{(1-A)R(\zeta, \tau_0)\zeta}{4+\left(3-\frac{1}{\tau_0}\int_0^{\tau_0} x_1(t)\,dt\right)(1-A)\tau_0}, \tag{4}$$

$$x_1(\tau) = \frac{3}{2}\int_0^\pi \chi(\gamma, \tau)\cos\gamma\sin\gamma\,d\gamma, \tag{5}$$

$$\bar{H}(\tau) = F - \frac{S}{4}e^{-\tau/\zeta}\zeta, \tag{6}$$

$$R(\zeta, \tau_0) = 1 + \frac{3}{2}\zeta + \left(1-\frac{3}{2}\zeta\right)e^{-\tau_0/\zeta}, \tag{7}$$

and the optical depth is

$$\tau(r) = \int_r^{R_A} \alpha(r)\,dr. \tag{8}$$

Just as in the case of a heterogeneous, purely scattering, atmosphere we have

$$B(\tau, \eta, \zeta, \gamma) = \Delta B(\tau, \eta, \zeta) + \frac{S}{4}\chi(\gamma, r)e^{-\tau/\zeta}. \tag{9}$$

where the function ΔB (τ, η, ζ) is found from Eq. (2).

So, converting to a suitable system of coordinates, and substituting Eq.(9) into Eq.(1), we obtain the unknown expression for the intensity of radiation determining the glow of the atmospheric layers on the limb side of the planet.

Let us suppose, for the sake of simplicity, that the line of sight, KB (Figure 1), lies in the plane of the planet's intensity equator. Now the position of this line with respect to the centers of planet and Sun will be determined by h and the scattering angle γ. Here h is the distance between the line of sight and the planet's surface. And for the current point T on the line of sight

$$s = \sqrt{R_A^2 - r_0^2} \mp \sqrt{r^2 - r_0^2}, \tag{10}$$

where $r_0 = R + h$. The sign is minus for the section BC, and plus for the section AC, of the line of sight. Then

$$ds = \mp \frac{r dr}{\sqrt{r^2 - r_0^2}}. \tag{11}$$

Readily found from Figure 1 are

$$\eta_{1,2} = \cos \varepsilon = \mp \frac{\sqrt{r^2 - r_0^2}}{r}, \tag{12}$$

$$\zeta_{1,2} = \cos i = \frac{r_0}{r} \sin \gamma \pm \frac{\sqrt{r^2 - r_0^2}}{r} \cos \gamma, \tag{13}$$

in which the subscripts 1 and 2 refer to the upper and lower signs in Eqs. (12) and (13), respectively. Substituting Eqs. (10) and (11) into Eq. (1), we obtain /58

$$I(h, \gamma) = \exp[-\beta(R_A)] \int_{r_0}^{R_A} \{B(\tau(r), \eta_1, \zeta_1) \exp[-\beta(r)] + B(\tau(r), \eta_2, \zeta_2) \exp[\beta(r)]\} \cdot \frac{d\beta(r)}{dr} dr, \tag{14}$$

where

$$\beta(r) = \int_{r_0}^{r} \alpha(r) \frac{r dr}{\sqrt{r^2 - r_0^2}}. \tag{15}$$

Eq. (14) is applicable only when the line of sight does not intersect the terminator. This condition is satisfied when angle $i \leq 90°$. With this in mind, and using Eq. (13), we find that interval of change in the scattering angle, γ, for which the line of sight does not intersect the terminator as

$$\arccos \frac{R}{R_A} \leqslant \gamma \leqslant \pi - \arccos \frac{R}{R_A}. \tag{16}$$

It should be noted that in practice inequality (16) can be strengthened such that the intersection of line of sight and terminator will take place only in the uppermost layers of the atmosphere where, for all practical purposes, solar radiation is not reradiated.

Finally, let us consider the case of an optically shallow atmosphere, ($\tau_0 \ll 1$), contiguous to a surface with low albedo ($A \ll 1$). Then we are correct in considering only first order scattering; that is, we can take the following expression for the source function

$$B(\tau, \eta, \zeta; \gamma) = \frac{S}{4}\chi(\gamma, r)\, e^{-\tau/\zeta}. \tag{17}$$

Substituting Eq. (17) into Eq. (14), and considering Eq. (13), after transformations, we obtain

$$I(h, \gamma) = \frac{S}{2}\exp\left[-\beta(R_A)\right]\int_{r_0}^{R_A} \chi(\gamma, r)\,\mathrm{ch}\left[\beta(r) + \frac{r\tau(r)\sqrt{r^2 - r_0^2}\cos\gamma}{r_0^2 - r^2\cos^2\gamma}\right]\exp\left[-\frac{rr_0\tau(r)\sin\gamma}{r_0^2 - r^2\cos^2\gamma}\right]\frac{d\beta(r)}{dr}\,dr. \tag{18}$$

This latter formula quite obviously is applicable to the Martian atmosphere (the visible region of the spectrum) as well as to a cloudless terrestrial atmosphere (in the long-wavelength region of the visible spectrum).

Let us now move on to the calculation of the brightness distribution along the limb side Martian atmospheric layers. Measurement of brightness distribution along atmospheric layers can yield valuable information on the vertical structure of the planet's atmosphere. In the case of Mars, Yung [10] attempted to use Mariner 4 observations to construct the brightness distribution along the atmospheric aureole. However, the sensitivity of the equipment carried by this space station was such that distribution for sufficiently large h of the line of sight above the planetary surface could not be obtained. Hence it is desirable to have rough a priori estimates of brightness distribution, for these will enable us to establish, in advance, the sensitivity of the equipment designed for such measurements. /59

It should be pointed out that we cannot, at this point, use Eq. (18) to arrive at sufficiently accurate calculations. The fact is that the distribution of aerosol particles with altitude in the Martian atmosphere is a practically unknown quantity, so that change in the scattering indicatrix and in the aerosol component, τ_a of the optical depth τ, is unknown. We have, therefore, made a number of simplifications, and more or less plausible assumptions, in order to arrive at very rough estimates.

A. V. Morozhenko has calculated [11] the total optical thickness of the Martian atmosphere to be $\tau_0 < 0.05$ for the spectral interval $\lambda \geq 0.4\mu$. This can result in the simplification of Eq. (18). Let us assume, for purposes of definiteness, that the geometric thickness of the Martian atmosphere is $z_0 = 100$ km. If we say that R = 3400 km, then $R_A = 3500$ km. Let us make the calculation for angles $30° \leq \gamma \leq 150°$. Then, from Eq. (13), $0.27 < \zeta \leq 1$, and since $\tau_0 < 0.05$, we can use the following expression instead of Eq. (17) to make the approximate calculation for brightness distribution

$$B(\tau, \eta, \zeta, \gamma) = \frac{S}{4} \chi(\gamma, r). \tag{19}$$

And instead of Eq. (18), we have

$$I(h, \gamma) = \frac{S}{2} \exp[-\beta(R_A)] \int_{r_0}^{R_A} \chi(\gamma, r) \operatorname{ch}[\beta(r)] \frac{d\beta(r)}{dr} dr. \tag{20}$$

As usual (see [6], for example) we assume

$$\chi(\gamma, r) = \frac{\alpha_R(r)}{\alpha(r)} \chi_R(\gamma) + \frac{\alpha_a(r)}{\alpha(r)} \chi_a(\gamma), \tag{21}$$

where

$$\chi_R(\gamma) = \frac{3}{4}(1 + \cos^2\gamma) - \tag{22}$$

is the Rayleigh scattering indicatrix, $\chi_a(\gamma)$ is the aerosol scattering indicatrix

$$\alpha(r) = \alpha_a(r) + \alpha_R(r), \tag{23}$$

and $\alpha_R(r)$ is the Rayleigh scattering factor, $\alpha_a(r)$ is the scattering factor for the aerosol particles. In accordance with Eq. (23), the total optical thickness, τ_0, of the atmosphere can be given in the form

$$\tau_0 = \tau_{0R} + \tau_{0a}. \tag{24}$$

Let us take it that in the atmosphere the aerosol, α_a, and Rayleigh, α_R, scattering factors change exponentially. Let us put (when $z_0 \gg H$ and $z_0 \gg H_a$)

$$\alpha_a(r) = \frac{\tau_{0a}}{H_a} \exp\left(-\frac{r-R}{H_a}\right), \tag{25}$$

$$\alpha_R(r) = \frac{\tau_{0R}}{H} \exp\left(-\frac{r-R}{H}\right), \tag{26}$$

where H and H_a are the altitudes of the homogeneous gas and aerosol components of the atmosphere, respectively.

Substituting Eq. (21) into Eq. (20), and giving consideration to Eqs. (15) /60
and (23) instead of Eq. (20) we find

$$I(h,\gamma)=\frac{S}{2}\exp[-\beta(R_A)]\left\{\chi_R(\gamma)\,\mathrm{sh}[\beta(R_A)]+\right.$$
$$\left.+[\chi_a(\gamma)-\chi_R(\gamma)]\int_{r_0}^{R_A}\mathrm{ch}[\beta(r)]\frac{\alpha_a(r)\,rdr}{\sqrt{r^2-r_0^2}}\right\}. \tag{27}$$

Considering Eq. (15), and the fact that ordinarily $H \gg H_a$, and $\tau_{0a} \gg \tau_{0R}$, the integral in the brackets in Eq. (27) can be given in the following approximate form

$$\int_{r_0}^{R_A}\mathrm{ch}[\beta(r)]\frac{\alpha_a(r)\,dr}{\sqrt{r^2-r_0^2}}\approx\int_{r_0}^{R_A}\mathrm{ch}\left[\int_{r_0}^{r}\alpha_a(r')\frac{r'\,dr'}{\sqrt{r'^2-r_0^2}}\right]\times$$
$$\times\frac{\alpha_a(r)\,rdr}{\sqrt{r^2-r_0^2}}=\mathrm{sh}\left[\int_{r_0}^{R_A}\frac{\alpha_a(r)\,rdr}{\sqrt{r^2-r_0^2}}\right]. \tag{28}$$

Using Eq. (26), we can write

$$\int_{r_0}^{R_A}\frac{\alpha_a(r)\,rdr}{\sqrt{r^2-r_0^2}}=\frac{\tau_{0a}}{H_a}e^{\frac{R}{H_a}}\left[r_0K_1\left(\frac{r_0}{H_a}\right)-\int_{R_A}^{\infty}\frac{e^{-\frac{r}{H_a}}rdr}{\sqrt{r^2-r_0^2}}\right], \tag{29}$$

where $K_1(z)$ is a cylindrical first order function of the imaginary argument.

It is obvious that for those r_0 values for which the magnitude

$$\frac{e^{-\frac{R_A-R}{H_a}}}{\sqrt{R_A^2-r_0^2}}\ll 1, \tag{30}$$

we can ignore the second summand in the right-hand side of Eq. (29) as compared to the first. If, moreover, it is considered that for $z \gg 1$ (see [12], p. 977)

$$K_1(z)\sim\sqrt{\frac{\pi}{2z}}e^{-z}\left[1+O\left(\frac{1}{z}\right)\right], \tag{31}$$

instead of Eq. (29), we obtain

$$y=\int_{r_0}^{R_A}\frac{\alpha_a(r)\,rdr}{\sqrt{r^2-r_0^2}}=\tau_{0a}\sqrt{\frac{\pi r_0}{2H_a}}e^{-\frac{h}{H_a}}. \tag{32}$$

as well as

$$x=\int_{r_0}^{R_A}\frac{\alpha_R(r)\,rdr}{\sqrt{r^2-r_0^2}}=\tau_{0R}\sqrt{\frac{\pi r_0}{2H}}e^{-\frac{h}{H}}. \tag{33}$$

Now we can estimate the relative error, $\Delta(h)$, in Eq. (28), providing the Eq. (30) condition is satisfied. Considering Eqs. (15) and (23), we find the following from Eq. (28), with the help of Eqs. (32) and (33)

$$\left\{\int_{r_0}^{R_A} \mathrm{ch}\left[\int_{r_0}^{r} \alpha_a(r')\frac{r'\,dr'}{\sqrt{r'^2-r_0^2}} + \int_{r_0}^{r}\alpha_R(r')\frac{r'\,dr'}{\sqrt{r'^2-r_0^2}}\right]\frac{\alpha_a(r)\,r\,dr}{\sqrt{r^2-r_0^2}} - \mathrm{sh}\left[\int_{r_0}^{R_A}\frac{\alpha_a(r)\,r\,dr}{\sqrt{r^2-r_0^2}}\right]\right\} \leqslant \left\{\int_{r_0}^{R_A}\mathrm{ch}\left[\int_{r_0}^{r}\alpha_a(r')\frac{r'\,dr'}{\sqrt{r'-r_0^2}} + \int_{r_0}^{\infty}\alpha_R(r)\frac{r\,dr}{\sqrt{r^2-r_0^2}}\right]\frac{\alpha_a(r)\,r\,dr}{\sqrt{r^2-r_0^2}} - \mathrm{sh}\,y\right\} = \mathrm{sh}\,(x+y)-\mathrm{sh}\,x-\mathrm{sh}\,y, \quad (34)$$ /61

from whence, after uncomplicated transformations, we obtain

$$\Delta(h) \leqslant \frac{2\,\mathrm{sh}\,\frac{x}{2}\,\mathrm{sh}\,\frac{x+y}{2}}{\mathrm{sh}\,\frac{y}{2}}. \quad (35)$$

Thus, by using Eqs. (28), (32), and (33) in place of Eq. (27) we obtain the following approximate formula for calculating the intensity of emergent radiation (for an atmosphere with a very small optical thickness)

$$I(h,\gamma) = \frac{S}{2}\exp\left[-\sqrt{\frac{\pi r_0}{2}}\left(\frac{\tau_{0R}}{\sqrt{H}}e^{-\frac{h}{H}} + \frac{\tau_{0a}}{\sqrt{H_a}}e^{-\frac{h}{H_a}}\right)\right]\times \left\{\chi_R(\gamma)\,\mathrm{sh}\left[\sqrt{\frac{\pi r_0}{2}}\left(\frac{\tau_{0R}}{\sqrt{H}}e^{-\frac{h}{H}} + \frac{\tau_{0a}}{\sqrt{H_a}}e^{-\frac{h}{H_a}}\right)\right] + [\chi_a(\gamma)-\chi_R(\gamma)]\,\mathrm{sh}\left(\tau_{0a}\sqrt{\frac{\pi r_0}{2H_a}}e^{-\frac{h}{H_a}}\right)\right\}. \quad (36)$$

The numerical values for $\chi_a(\gamma)$, τ_{0a} and τ_{0R} used for the calculations involving Eq. (36) were taken from A. V. Morozhenko [11]. These data are listed in the table. Further, as per [10], H = 12, H_a = 3 km. Eq. (36) calculations were made for S = 1, and the results are shown in Figures 2-4. We should note that $\Delta(h)$ < 16 percent for the numerical values of the optical parameters used [see inequality (35)].

NUMERICAL VALUES OF $\chi_a(\gamma)$, τ_{Oa}, and τ_{OR} /62

γ	$\tilde{\lambda}_\mu$		
	0.4	0.5	0.6
	$\chi_a(\gamma)$	$\chi_a(\gamma)$	$\chi_a(\gamma)$
30°	3.30	3.27	3.24
45	1.89	1.88	1.87
60	1.06	1.06	1.06
90	0.435	0.437	0.440
120	0.321	0.327	0.333
150°	0.385	0.394	0.403
τ_{0R}	0.0100	0.0041	0.0019
τ_{0a}	0.039	0.026	0.019

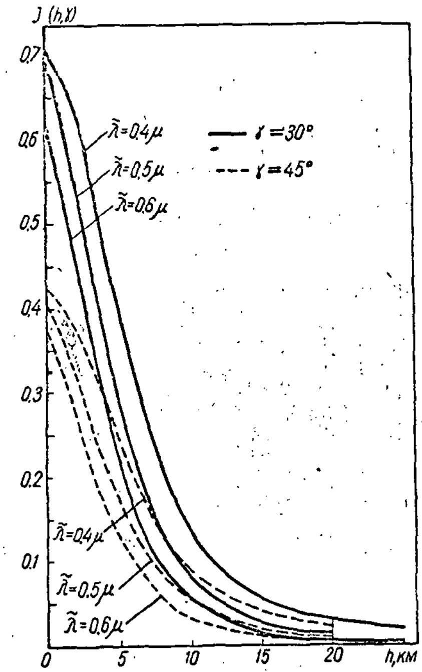

Figure 2.

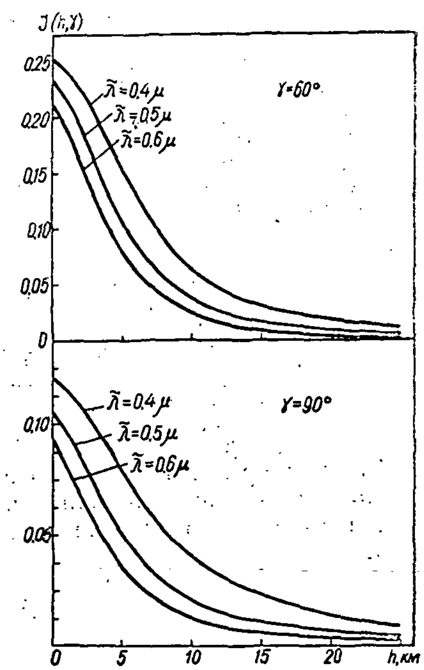

Figure 3.

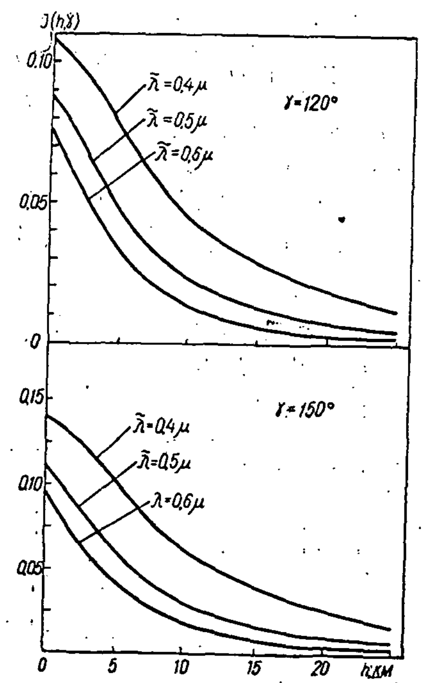

Figure 4.

As will be seen from Figures 2-4, the layers of the atmosphere at the Martian surface with the brightest glow are some 10 km thick. Moreover, as should have been expected, the brightness of the atmospheric layers at a specified altitude, h, is greater the smaller the angle of scatter, γ, all other conditions being equal. This latter evidently is associated with the extensive elongation of the aerosol scattering indicatrix.

In the future, as our concepts of the vertical structure of the Martian atmosphere are refined, it will make sense to repeat the calculations using the more precise Eq. (14). In the meantime, we obviously will have to limit ourselves to the results shown in Figures 2-4 in order to obtain rough estimates.

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